



Integration Exercises

Exercise 1. Determine the antiderivatives of $\cosh$ and $\sinh$.

↑ primitive

$$\int \cosh(x) dx = \int \frac{e^x + e^{-x}}{2} dx = \frac{1}{2} \left(\int e^x dx + \int e^{-x} dx \right) = \frac{1}{2} (e^x + c_1 + (-e^{-x}) + c_2) =$$

$$= \underbrace{\frac{e^x - e^{-x}}{2}}_{\sinh(x)} + \underbrace{\frac{c_1 + c_2}{2}}_{c_3 \in \mathbb{R}} = \sinh(x) + c_3 \quad \begin{array}{l} (e^x)' = e^x \quad (e^{-x})' = e^{-x}(-1) \quad c_1, c_2 \in \mathbb{R} \\ (-e^{-x})' = e^{-x} \end{array}$$

$$\int \sinh(x) dx = \int \frac{e^x - e^{-x}}{2} dx = \frac{1}{2} \left(\int e^x dx - \int e^{-x} dx \right) = \frac{1}{2} (e^x + e^{-x} + c) = \underbrace{\frac{e^x + e^{-x}}{2}}_{\cosh(x)} + \underbrace{c}_{\in \mathbb{R}} =$$

$$= \cosh(x) + c, c \in \mathbb{R}$$