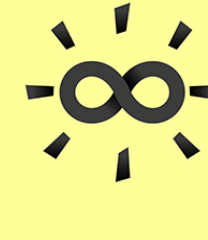


The Bright Side of Mathematics



Fourier Series

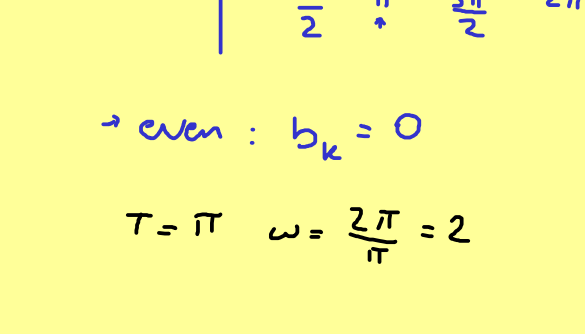
Exercises 1

Exercise 1. Compute the Fourier series of $f(x) = |\sin(x)|$.

$$f(x) \approx \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(k\omega x) + b_k \sin(k\omega x)) \quad \omega = \frac{2\pi}{T}$$

$$a_k = \frac{2}{T} \int_0^T f(x) \cos(k\omega x) dx, \quad k \geq 0$$

$$b_k = \frac{2}{T} \int_0^T f(x) \sin(k\omega x) dx, \quad k \geq 1$$



→ even : $b_k = 0$

$$T = \pi \quad \omega = \frac{2\pi}{\pi} = 2$$

$$\frac{a_0}{2} = \frac{1}{\pi} \int_0^{\pi} \sin(x) dx = \frac{1}{\pi} (-\cos\pi + \cos 0) = \frac{2}{\pi}$$

$$a_k = \frac{2}{\pi} \int_0^{\pi} \sin(x) \cos(2kx) dx$$

$$\int \sin(x) \cos(2kx) dx = -\cos(x) \cos(2kx) - 2k \int \cos(x) \sin(2kx) dx$$

$$f(x) = \sin(x) \quad g(x) = \cos(2kx)$$

$$f'(x) = \cos(x) \quad g(x) = \sin(2kx)$$

$$f(x) = -\cos(x) \quad g'(x) = -\sin(2kx) 2k$$

$$f(x) = \sin(x) \quad g'(x) = \cos(2kx) 2k$$

$$\int \sin(x) \cos(2kx) dx = -\cos(x) \cos(2kx) - 2k \left(\sin(x) \sin(2kx) - 2k \int \sin(x) \cos(2kx) dx \right)$$

$$(1 - 4k^2) \int_0^{\pi} \sin(x) \cos(2kx) dx = (-\cos(x) \cos(2kx) - 2k \sin(x) \sin(2kx)) \Big|_0^{\pi}$$

$$\int_0^{\pi} \sin(x) \cos(2kx) dx = \frac{1}{1-4k^2} \left(-(-1)(1) - (-(-1)(1)) \right) = \frac{2}{1-4k^2}$$

$$a_k = \frac{2}{\pi} \cdot \frac{2}{1-4k^2} \quad k \geq 1, \quad \frac{a_0}{2} = \frac{2}{\pi}, \quad b_k = 0$$

$$f(x) \approx \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(k\omega x) + b_k \sin(k\omega x))$$

$$|\sin(x)| \approx \frac{2}{\pi} + \sum_{k=1}^{\infty} \frac{4}{\pi(1-4k^2)} \cos(2kx)$$